IMPROVEMENT AND VALIDATION OF A MODEL FOR PHOTOVOLTAIC ARRAY PERFORMANCE

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ABSTRACT

The 5-Parameter model described in this paper uses data provided by the manufacturer, together with semi-empirical correlation equations and weather data, to predict annual energy production. Annual data obtained from a building integrated photovoltaic facility at the National Institute of Standards and Technology (NIST) were provided for four different cell technologies. The 5-Parameter model and the King [1] model are compared with the measured data provided by NIST over a wide range of operating conditions. Both models show excellent results.

1. INTRODUCTION

The electrical power output from a photovoltaic panel depends primarily on the incident solar radiation, the cell temperature, and solar incidence angle. Manufacturers typically provide only limited operational data for photovoltaic panels that include: open circuit voltage (V_{oc}), the short circuit current (I_{sc}), the maximum power current (I_{mp}) and voltage (V_{mp}), the temperature coefficients at open circuit voltage and short circuit current (β_{Voc} and α_{Isc}, respectively), and the nominal operating cell temperature (NOCT). These data are available only at Standard Rating Conditions (SRC), for which the irradiance is 1,000 W/m^2 and the cell temperature (T_c) is 25°C (except for the NOCT which is determined at 800 W/m^2, 20°C). The purpose of this paper is to demonstrate how the performance information provided at SRC can be used to determine the photovoltaic system performance at any specified condition.

2. CURRENT-VOLTAGE RELATIONSHIP

The electrical power available from a photovoltaic (PV) device can be modeled with the circuit shown in Fig. 1 [2, 3].

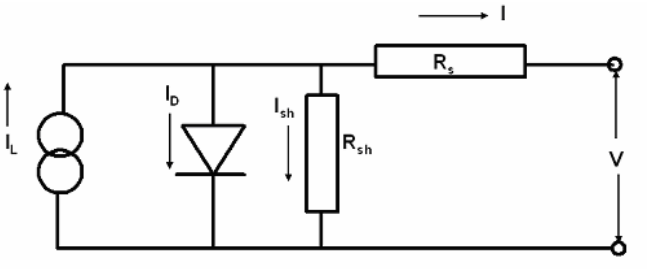


Fig. 1: Equivalent circuit for the 5-Parameter model

Five parameters must be known in order to determine the current and voltage, and thus the power delivered to the load. These are: the light current I_L, the diode reverse saturation current I_o, the series resistance R_s, the shunt resistance R_{sh}, and a modified ideality factor $a \equiv N_s n_I k T_c/q$. The electron charge q, and Boltzmann's constant k are known, n_I is the usual ideality factor, N_s is the number of cells in series and T_c is the cell temperature. The current-voltage relationship at a fixed cell temperature and solar radiation for the circuit in Fig. 1 is expressed in Eqn. (1).

$$I = I_L - I_o\left[e^{\frac{V+IR_s}{a}} - 1\right] - \frac{V+IR_s}{R_{sh}} \qquad (1)$$

3. CALCULATING THE REFERENCE PARAMETERS

To evaluate the five parameters in Eqn. (1), five independent pieces of information are needed. In general, these five parameters are functions of the solar radiation incident on the cell and cell temperature. Reference values of these parameters are determined for a specified operating condition such as SRC. Three current-voltage pairs are normally available from the manufacturer at SRC: the short circuit current, the open circuit voltage and the current and

voltage at the maximum power point. A fourth piece of information results from recognizing that the derivative of the power at the maximum power point is zero. Although both the temperature coefficient of the open circuit voltage (β_{Voc}) and the temperature coefficient of the short circuit current (α_{Isc}) are known, only β_{Voc} is used to find the five reference parameters. α_{Isc} is used when the cell is operating at conditions other than reference conditions.

Substituting the known information into Eqn, (1) results into Eqns. (2-7) which determine the circuit parameters (a_{ref}, $I_{o,ref}$, $I_{L,ref}$, $R_{s,ref}$, *and* $R_{sh,ref}$) at reference conditions.

For short circuit current: $I=I_{sc,ref}$, $V=0$

$$I_{sc,ref} = I_{L,ref} - I_{o,ref}\left[e^{\frac{I_{sc,ref}R_{s,ref}}{a_{ref}}} - 1\right] - \frac{I_{sc,ref}R_{s,ref}}{R_{sh,ref}} \tag{2}$$

For open circuit voltage: $I=0$, $V=V_{oc,ref}$

$$0 = I_{L,ref} - I_{o,ref}\left[e^{\frac{V_{oc,ref}}{a_{ref}}} - 1\right] - \frac{V_{oc,ref}}{R_{sh,ref}} \tag{3}$$

At the maximum power point: $I=I_{mp,ref}$, $V=V_{mp,ref}$

$$I_{mp,ref} = I_{L,ref} - I_{o,ref}\left[e^{\frac{V_{mp,ref}+I_{mp,ref}R_{s,ref}}{a_{ref}}} - 1\right] - \frac{V_{mp,ref}+I_{mp,ref}R_{s,ref}}{R_{sh,ref}} \tag{4}$$

At the maximum power point,

$$\left.\frac{d(IV)}{dV}\right|_{mp} = I_{mp} - V_{mp}\left.\frac{dI}{dV}\right|_{mp} = 0. \tag{5}$$

where $\left.dI/dV\right|_{\mathrm{mp}}$ is given by:

$$\left.\frac{dI}{dV}\right|_{mp} = \frac{\frac{-I_o}{a}e^{\frac{V_{mp}+I_{mp}R_s}{a}} - \frac{1}{R_{sh}}}{1+\frac{I_oR_s}{a}e^{\frac{V_{mp}+I_{mp}R_s}{a}} + \frac{R_s}{R_{sh}}} \tag{6}$$

The temperature coefficient of open circuit voltage is given by:

$$\beta_{Voc} = \left.\frac{\partial V}{\partial T}\right|_{I=0} \approx \frac{V_{oc,ref} - V_{oc,T_c}}{T_{ref} - T_c} \tag{7}$$

To evaluate β_{Voc} numerically, it is necessary to know V_{oc,T_c}, the open circuit voltage at some cell temperature near the reference temperature. The cell temperature used for this purpose is not critical since values of T_c ranging from 1 to 10 K above or below T_{ref} provide essentially the same result. In order to apply Eqn. (7), it is necessary to obtain expressions for the temperature dependence of the three parameters a, I_o and, I_L. The temperature dependence of all of the parameters in the model is considered in the following section.

4. DEPENDENCE OF OPERATING CONDITIONS

From the definition of a, the modified ideality factor is a linear function of cell temperature (assuming n_I is independent of temperature) so that:

$$\frac{a}{a_{ref}} = \frac{T_c}{T_{c,ref}} \tag{8}$$

where $T_{c,ref}$ and a_{ref} are the cell temperature and modified ideality factor for reference conditions, while T_c and a are the cell temperature and modified ideality factor parameter for the new operating conditions.

Messenger and Ventre [4] present an equation from diode theory for the diode reverse saturation current, I_o. The ratio of their equation at the new operating temperature to that at the reference temperature yields:

$$\frac{I_o}{I_{o,ref}} = \left[\frac{T_c}{T_{c,ref}}\right]^3 \exp\left[\frac{1}{k}\left(\left.\frac{E_g}{T}\right|_{T_{ref}} - \left.\frac{E_g}{T}\right|_{T_c}\right)\right] \tag{9}$$

where k is Boltzmann's constant and E_g is the material band gap. The values of the material band gap energies at 25°C for the four cell types investigated in this study can be found in Table A1. E_g exhibits a small temperature dependence [5] which, for silicon, can be represented as indicated in Eqn. (10) where $E_{g,T_{ref}}$ =1.121 eV for silicon cells. Equation (10) was used for all of the cells considered in this study although the value of $E_{g,T_{ref}}$ used for the triple junction amorphous cell type was 1.6 eV.

$$\frac{E_g}{E_{g,T_{ref}}} = 1 - 0.0002677\left(T - T_{ref}\right) \tag{10}$$

The light current, (I_L), is nearly a linear function of incident solar radiation. Some pyranometers in fact use the short circuit current of a solar cell as a measure of the incident solar radiation. The light current (I_L) is observed to depend on the absorbed solar irradiance (S), the cell temperature (T_c), the short circuit current temperature coefficient (α_{Isc}), and the air mass modifier (M). The light current I_L for any operating conditions is related to the light current at reference conditions by:

$$I_L = \frac{S}{S_{ref}}\frac{M}{M_{ref}}[I_{L,ref} + \alpha_{Isc}(T_c - T_{c,ref})] \tag{11}$$

where S_{ref}, $M_{ref,}$ $I_{L,ref}$, $T_{c,ref}$ are the parameters at reference conditions, while S, M, I_L, and T_c are the values for specified operating conditions. When using Eqn. (11) to find the reference parameters, $S = S_{ref}$ and $M = M_{ref}$. The air mass modifier is assumed to be a function of the local zenith angle and is discussed below.

The information needed to determine the reference parameters is now complete. Eqns. (3-7) relate the model to the known reference conditions. To evaluate Eqn. (7) it is necessary to use the temperature dependence of a, I_o and I_L as given by Eqns. (8-11).

The final task to complete the model is to investigate the operating condition dependence of the series resistance R_s, and the shunt resistance, R_{sh}. The series resistance impacts the shape of current and voltage curve near the maximum power point. The effect on the I-V curve is small and, although methods of adjusting R_s as a function of operating conditions have been investigated [6], R_s is assumed constant at its reference value, $R_{s,ref}$ in this study.

The shunt resistance (R_{sh}) controls the slope of the I-V curve at the short circuit condition; large shunt resistances result in a horizontal slope. The shunt resistance appears to change with absorbed solar radiation for all of the cells although the effect is most noticeable for cell types that have a relatively small shunt resistance at SRC. If experimental data were generally available at more than one solar radiation value, it would be possible to develop a relation between the shunt resistance and absorbed radiation. However, this information is not normally available. An observation apparent from an examination of the slopes of the I-V curves at short circuit conditions based on the experimental data is that the effective shunt resistance increases (and the slope thus decreases) as absorbed radiation is reduced. Eqn (12) is proposed to describe this effect.

$$\frac{R_{sh}}{R_{sh,ref}} = \frac{S_{ref}}{S} \tag{12}$$

5. THE INCIDENCE ANGLE MODIFIER, $K_{\tau\alpha}$

The incidence angle θ is directly involved in the determination of the radiation incident on the surface of the PV device. In addition, the incidence angle affects the amount of solar radiation transmitted through the protective cover and absorbed by the cell. As the incidence angle increases, the amount of radiation reflected from the cover increases with the most noticeable effects occurring at incidence angles greater than 65°.

The effects of reflection and absorption as a function of incidence angle are expressed in terms of the incidence angle modifier, $K_{\tau\alpha}(\theta)$, defined as the ratio of the radiation absorbed by the cell at some incidence angle θ divided by radiation absorbed by the cell at normal incidence. The incidence angle can be calculated as a function of the panel slope and location and time. Panels that are mounted on a vertical surface exacerbate the incidence angle effects because much of the available beam solar radiation strikes the panel surface at angles greater than 65°. Nevertheless, vertically mounted panels are of interest because of the applicability of this orientation to installation on building façades. The experimental data that are available to validate the model presented in this paper were taken on a vertical surface.

The incidence angle modifier for a PV panel differs somewhat from that of a flat-plate solar collector in that the glazing may be bonded to the cell surface, thereby eliminating one air-glazing interface and the glazing surface may be treated so as to reduce reflection losses. Equations (13) and (14) based on Snell's and Bougher's laws, as reported in Duffie and Beckman [2], are used to calculate the incidence angle modifier. The angle of refraction (θ_r) is determined from Snell's law

$$\theta_r = \arcsin\left(n \sin\theta\right) \tag{13}$$

where θ is the incidence angle and n is an effective index of refraction of the cell cover. A good approximation of the transmittance of the cover system is:

$$\tau(\theta) = e^{-(KL/\cos\theta_r)}\left[1 - \frac{1}{2}\left(\frac{\sin^2(\theta_r - \theta)}{\sin^2(\theta_r + \theta)} + \frac{\tan^2(\theta_r - \theta)}{\tan^2(\theta_r + \theta)}\right)\right] \tag{14}$$

where K is the glazing extinction coefficient and L is the glazing thickness. In this study the value of K is assumed to be 4 m^{-1}, the value for "water white" glass and the glazing thickness is assumed to be 2 mm, a reasonable value for most PV cell panels. The refractive index is set to 1.526, the value for glass.

To obtain the incidence angle modifier ($K_{\tau\alpha}$), Equation (15) needs be evaluated for incidence angles of 0° and θ. The ratio of these two transmittances yields $K_{\tau\alpha}$:

$$K_{\tau\alpha}(\theta) = \frac{\tau(\theta)}{\tau(0)} \tag{15}$$

Separate incidence angle modifiers are needed for beam, diffuse, and ground-reflected radiation, but each can be calculated in the same way. Average angles for isotropic diffuse and ground-reflected radiation are provided as a function of the slope of the panel by Fig. 5.4.1 of Duffie and Beckman [2]. Although these average angles for diffuse radiation were obtained for thermal collectors, they were found to yield reasonable results for PV systems.

6. THE AIR MASS MODIFIER, M

Air mass is the ratio of the mass of air that the beam radiation has to traverse at any given time and location to

the mass of air that the beam radiation would traverse if the sun were directly overhead. Selective absorption by species in the atmosphere causes the spectral content of irradiance to change, altering the spectral distribution of the radiation incident on the PV panel. King et al. [7] developed an empirical relation to account for changes in the spectral distribution resulting from changes in air mass:

$$\frac{M}{M_{ref}} = a_0 + a_1 AM + a_2 AM^2 + a_3 AM^3 + a_4 AM^4 \qquad (16)$$

In Equation (16), AM is the air mass and a_o, a_1, a_2, a_3, and a_4 are constants for different PV materials. These constants are listed in Table A1, along with other values used for this study. However, if one set of constants is chosen and used for all cell types, the difference in the results obtained is insignificant so that the values of a_i for the poly-crystalline cell in Table A1 can be used for all cases.

7. CALCULATING THE ABSORBED RADIATION, S

The major factor affecting the power output from a PV device is the solar radiation absorbed on the cell surface, S, which is a function of the amount of radiation and the incidence angle. Radiation data are not normally known on the plane of the PV panel, so it is necessary to estimate the absorbed solar radiation using horizontal data and incidence angle information. The total absorbed irradiance S consists of beam, diffuse, and ground reflected components. The beam radiation G_b is the solar radiation received from the sun that has not been scattered by the atmosphere while the diffuse radiation G_d is the total horizontal radiation G minus the beam radiation. The required beam-diffuse breakup was done using the Erbs correlation [2]. Equation (11) provides a means of estimating the absorbed radiation, S, assuming that both diffuse and ground-reflected radiation are isotropic [2].

$$\frac{S}{S_{ref}} = \frac{G_b}{G_{ref}} R_{beam} K_{\tau\alpha,b} + \frac{G_d}{G_{ref}} K_{\tau\alpha,d} \frac{(1+Cos\beta)}{2} + \frac{G}{G_{ref}} \rho K_{\tau\alpha,g} \frac{(1-Cos\beta)}{2} \qquad (11)$$

In Equation (11), ρ is the ground reflectance, β is the slope of the panel, $K_{\tau\alpha,b}$ is the incidence angle modifier at the beam incidence angle, $K_{\tau\alpha,d}$ and $K_{\tau\alpha,g}$ are the incidence angle modifiers at effective incidence angles for isotropic diffuse and ground-reflected radiation, and R_{beam} is the ratio of beam radiation on a tilted surface to that on a horizontal.

8. VALIDATION OF THE 5-PARAMETER MODEL

The data used for this study were provided by Fanney et al. [8] from a building integrated photovoltaic facility at NIST in Gaithersburg, Maryland. The experimental data provide at five-minute intervals one year (Jan 1-Dec. 31, 2000) of meteorological data and measured cell temperature along with current and voltage values for four different photovoltaic cell technology types (single crystalline, poly-crystalline, silicon thin film, and 3-junction amorphous) installed on a vertical surface.

King [1] has developed a model to predict the energy production from a PV array. This model determines the current at five strategically located points on the current-voltage (I-V) curve and uses this information to identify the entire I-V curve. The accuracy of the King model is excellent. However, its disadvantage is that it requires a large amount of experimental data.

Figures 3-6 show results obtained with the 5-Parameter and King models. Note that the reference parameters were determined at the SRC condition, 1,000 W/m^2 and 25°C.

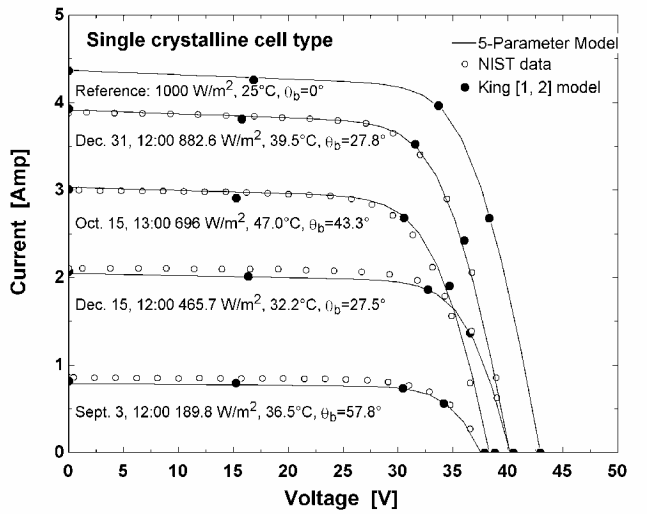


Fig. 3: Current vs voltage for the single-crystalline cell type.

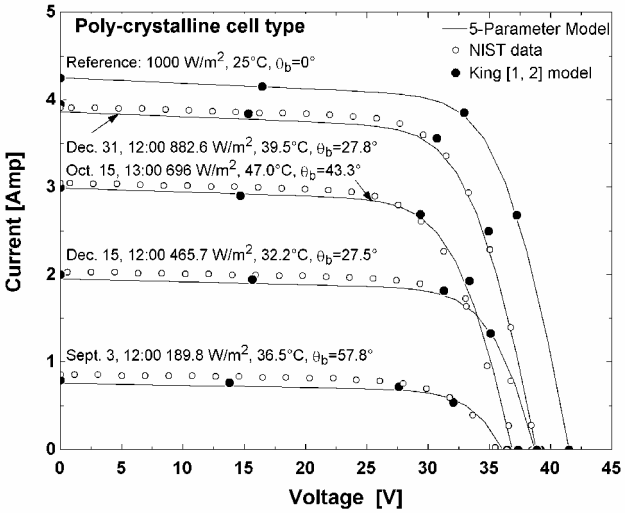


Fig. 4: Current vs voltage for the poly-crystalline cell type.

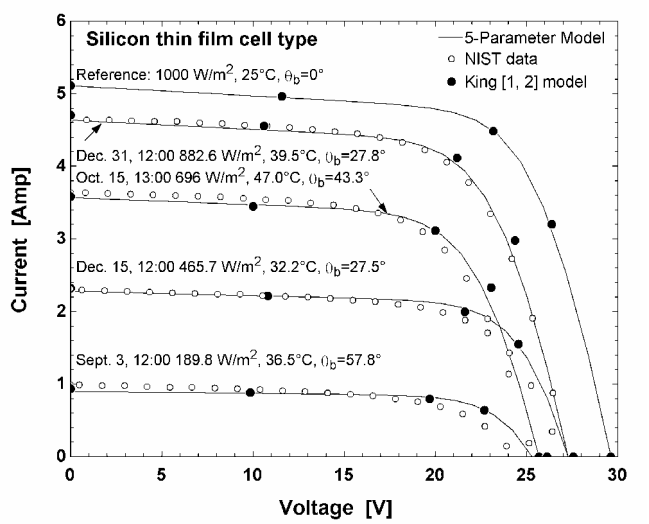


Fig. 5**:** Current vs voltage for the silicon thin film cell type.

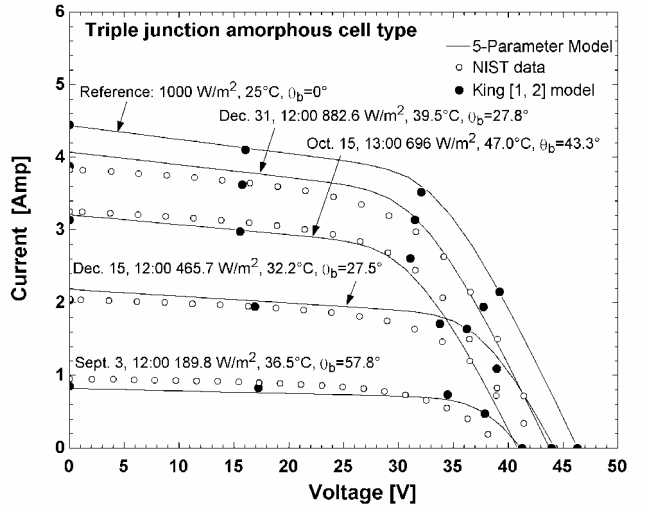


Fig. 6**:** Current vs voltage for the amorphous triple junction cell type.

Figures 3-6 show very good agreement for both the 5-Parameter model and the King model with the experimental data for all four cell types over a range of operating conditions.

9. CONCLUSIONS

The 5-Parameter model presented in this paper uses only data provided by the manufacturer with semi-empirical equations to predict the cell *I-V* curve (and thus the power) for any operating condition. The model requires a one-time calculation of the five parameters (a_{ref}, $I_{o,ref}$, $I_{L,ref}$, $R_{s,ref}$, and $R_{sh,ref}$) at reference conditions. These values are then used with in the model to calculate the parameters at other operating conditions, making it possible to predict the power output at any operating conditions. The shunt resistance was observed to be a function of absorbed radiation and an empirical equation was proposed to describe this effect.

Comparisons with experimental data provided by NIST [8] have shown that the 5-Parameter model can be an accurate tool for the prediction of energy production for single-junction cell types. The 5-Parameter model uses only data provided by the manufacturer, and in contrast to the King model, does not require parameters that need to be predetermined by additional experiments. The predictions from the 5-Parameter model are shown to agree well with both the King model results and the NIST [8] measurements for all four cell types over a range of operating conditions. The differences between the experimental data and the 5-Parameter model could be reduced if additional experimental data, e.g., I-V curves at two radiation levels, were used to determine the reference parameters.

10. ACKNOWLEDGMENTS

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Table A1: Values provided by NIST for the different cell types

Type of Cell	Silicon thin film	Single-crystalline	Poly-crystalline	3-Junction amorphous
At Reference Conditions				
$P_{mp,ref}$ (W)	103.96	133.40	125.78	57.04
$I_{sc,ref}$ (A)	5.11	4.37	4.25	4.44
$V_{oc,ref}$ (V)	29.61	42.93	41.50	23.16
$I_{mp,ref}$ (A)	4.49	3.96	3.82	3.61
$V_{mp,ref}$ (V)	23.17	33.68	32.94	16.04
NOCT (K)	316.15	316.85	316.45	311.05
Temperature Coefficients				
α_{Isc} (A/K)	0.00468	0.00175	0.00238	0.00561
α_{Isc} (1/K)	0.000916	0.000401	0.000560	0.001263
α_{Imp} (A/K)	0.00160	-0.00154	0.00018	0.00735
α_{Imp} (1/K)	0.000358	-0.000390	0.000047	0.002034
β_{Voc} (V/K)	-0.12995	-0.15237	-0.15280	-0.09310
β_{Voc} (1/K)	-0.004388	-0.003549	-0.003682	-0.004021
β_{Vmp} (V/K)	-0.13039	-0.15358	-0.15912	-0.04773
β_{Vmp} (1/K)	-0.005629	-0.004560	-0.004830	-0.002976
King Model Parameters Determined by NIST [9]. c_4 through c_7 were obtained from Sandia http://www.sandia.gov/pv/docs/Database.htm				
a_0	0.938110	0.935823	0.918093	1.10044085
a_1	0.062191	0.054289	0.086257	-0.06142323
a_2	-0.015021	-0.008677	-0.024459	-0.00442732
a_3	0.001217	0.000527	0.002816	0.000631504
a_4	-0.000034	-0.000011	-0.000126	-1.9184E-05
b_0	0.998980	1.000341	0.998515	1.001845
b_1	-0.006098	-0.005557	-0.012122	-0.005648
b_2	8.117E-04	6.553E-04	1.440E-03	7.25E-04
b_3	-3.376E-05	-2.730E-05	-5.576E-05	-2.916E-05
b_4	5.647E-07	4.641E-07	8.779E-07	4.696E-07
b_5	-3.371E-09	-2.806E-09	-4.919E-09	-2.739E-09
c_0	0.9615	0.9995	1.0144	1.072
c_1	0.0368	0.0026	-0.0055	-0.098
c_2	0.2322	-0.5385	-0.3211	-1.8457
c_3	-9.4295	-21.4078	-30.2010	-5.1762
c_4	0.967	0.9980	0.9931	1.059
c_5	0.033	0.0020	0.0069	-0.059
c_6	1.12	1.159	1.104	1.188
c_7	-0.120	-0.159	-0.104	-0.188
n_D	1.357	1.026	1.025	3.09
Other Parameters				
N_s	40	72	72	22
E_g (eV) at 25°C	1.12	1.12	1.14	1.6